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AMPLIFICATION OF QUANTUM MESON MODES
IN THE LATE TIME OF THE CHIRAL PHASE TRANSITION

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We investigate the time evolution of the quantum meson modes in the late time of
chiral phase transition. In particular, it is shown that there exists a possible solution
to the equation of motion for the quantum meson modes, which reveals a parametric
resonance and/or resonance through forced oscillation induced by the small oscillation of
the chiral condensate. After that, we demonstrate the unstable regions for the quantum
meson modes in both the cases of a uniform and spatially expanding system.

1. Introduction

In the experience of ultra-relativistic high energy nucleus-nucleus collision, it is
concerned that chiral symmetry is restored. However, when it is cooled down im-
mediately, spontaneous symmetry breaking occurs again. When chiral condensate
returns to actual vacuum, the amplitude of quantum pion modes with low mo-
menta even in the late time of chiral phase transition may grow. This phenomena
may connect to the parametric resonance.

In this paper, we are going to introduce the other possibility which may exist
to this mechanism, especially we would like to focus on the possibility by forced
oscillation. To investigate the dynamical chiral phase transition, we use the vari-
ational approach to the O(4) linear sigma model in the Gaussian wave functional
approximation.[1,2] It is shown that the amplification occurs in the mechanism of
the resonance by forced oscillation as well as the parametric resonance induced by
the small oscillation of the chiral condensate.[3]

2. TDVA with Gaussian wave functional

In this section, we find the equations of motion for the chiral condensate and quan-
tum fluctuations. In order to treat the chiral condensate and the fluctuation modes
around it self-consistently, we use the time-dependent variational approach (TDVA)

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with a Gaussian wave functional based on a functional Schrodinger picture. We start
with the following Hamiltonian,

$$\hat{H} = \int d^3\boldsymbol{x} \left\{ \frac{1}{2}\pi_a^2(\boldsymbol{x}) + \frac{1}{2}(\nabla\varphi_a(\boldsymbol{x}))^2 + \frac{m^2}{2}\varphi_a(\boldsymbol{x})^2 + \frac{\lambda}{24}(\varphi_a(\boldsymbol{x})^2)^2 - h\varphi_0(\boldsymbol{x}) \right\}, (1)$$

where a runs from 0 to 3. The index 0 indicates the sigma field and $1-3$ indicate
the pion fields. The small h means explicit chiral symmetry breaking term. There
are three model parameters used here, i.e., m, λ and h. The model parameters m^2,
λ and h are taken so as to reproduce the pion mass $M_\pi = 138$ MeV, the sigma
meson mass $M_0 = 500$ MeV and the pion decay constant $f_\pi(= \varphi_0) = 93$ MeV in
vacuum, explicitly, $m^2 = -(519.70 \text{ MeV})^2$, $\lambda = 84.96$ and $h = (78.13 \text{ MeV})^3$.

We adopt the Gaussian wave functional as the trial wave functional in the
framework of the functional Schrödinger picture. The time-dependence of varia-
tional functions is governed by the time-dependent variational principle:

$$\delta \int dt \left\langle i\frac{\partial}{\partial t} - \hat{H} \right\rangle = 0 . \tag{2}$$

As the result, we obtain the following equations of motion:

$$\left\{ \partial_t^2 + M_0^2(t) - \frac{\lambda}{3}\bar{\varphi}_0(t)^2 \right\} \bar{\varphi}_0(t) = h , \tag{3}$$

$$\left\{ \partial_t^2 + \boldsymbol{k}^2 + M_a^2(t) \right\} u_a^{\boldsymbol{k}}(t) = 0 . \tag{4}$$

These form a set of basic equations of motion for the chiral condensate and quantum
meson fields. The $\bar{\varphi}_0$ can be regarded as chiral condensate and the $u_a^{(n)}$ can be
regarded as mode functions for quantum meson modes.

We now demonstrate qualitatively the time evolution of the mean field $\bar{\varphi}_a$ and
of the quantum meson mode functions $u_a^{(n)}$ in the case of a uniform system without
spatially expansion. We assume that $\bar{\varphi}_0 \neq 0$ and $\bar{\varphi}_i = 0$, with $i = 1 - 3$, that is,
the chiral condensate points in the sigma direction. In the numerical calculation,
we adopt the box normalization with length L in each direction. We then impose
periodic boundary conditions for the fluctuation modes, namely, the allowed values
of momenta are $k_x = (2\pi/L)n_x$ and so on, where n_x is an integer. The fluctuation
modes labeled by (n_x, n_y, n_z) are included in each direction up to $n^2 \equiv n_x^2 + n_y^2 + n_z^2 = 8^2$. This corresponds to the three momentum cutoff $\Lambda \sim 1$GeV (more precisely,
990 MeV), since we have adopted the collisional region as $L^3 = (10 \text{ fm})^3$. Further,
we assume that the fluctuation modes of the pion fields are all identical, and we
denote them $u_1^{\boldsymbol{k}} = u_2^{\boldsymbol{k}} = u_3^{\boldsymbol{k}} \equiv u_\pi^{\boldsymbol{k}}$.

In Fig. 1, the time evolution of the chiral condensate is depicted. In order to avoid
the complexity about problems of the initial conditions in relativistic heavy ion
collisions, we only demonstrate the time evolution qualitatively with $\bar{\varphi}_0(t = 0) = 40$
MeV and $\dot{\bar{\varphi}}_0(t = 0) = 0$. The reason we adopted the above initial conditions is that
we are interested in the behavior of the time evolution of the chiral condensate and
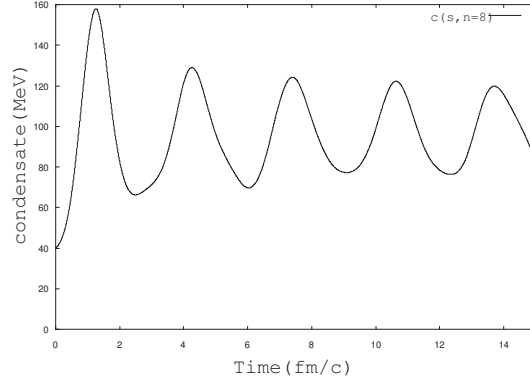quantum meson modes in the late time of the dynamical chiral phase transition

Fig. 1. The time evolution of the chiral condensate. The horizontal and vertical axes represent time and the value of the chiral condensation, respectively.
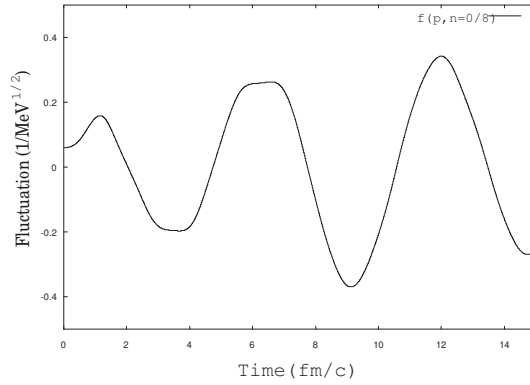

Fig. 2. The time evolution of the fluctuation mode with $n = 0$ in the π-direction.

in this paper. It is shown that the chiral condensate approaches the vacuum value with oscillation.

In Fig. 2, it is shown that the amplitude of the quantum pion mode with $n = 0$ is amplified. Similarly, the amplitude of the pion mode with $n = 1$ is also amplified weakly. However, amplification is not realized for modes with $n \geq 2$. By contrast, the sigma meson modes with $n \geq 0$ are not amplified.

Thus, we conclude that, in this parameterization, the amplitudes of the lowest and first excited pion modes are amplified, but other quantum meson modes are not.

The time evolution of the chiral condensate with the expanding geometry is also calculated. It is seen that the damped oscillational behavior is realized more

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quickly than in the case without spatial expansion.

It should be noted here that the amplification occurs in the late time of the chiral phase transition in both the cases of no spatial expansion and one-dimensional spatial expansion. It is thus seen that, even when the chiral condensate oscillates around its vacuum value with small amplitude, there are amplified solutions of pion modes. The mechanism of the amplification is clarified in the next section.

3. Late time of chiral phase transition

In this section, we investigate the mechanism of the amplification for meson modes. It is shown that one of possible mechanisms is a forced oscillation, as well as parametric resonance.

Let us investigate the time-dependent solutions around the static configurations. The condensate and quantum meson modes can be expanded around the static solutions.

$$\bar{\varphi}_0(t) = \varphi_0 + \delta\varphi(t) \; ,$$
$$u_a^{\mathbf{k}}(t) = u_a^{\mathbf{k}^s}(t) + \delta u_a^{\mathbf{k}}(t) \; . \tag{5}$$

Here, we consider the late time of the chiral phase transition. Then, $|\delta\varphi(t)|$ is small compared with the vacuum value, φ_0. Further, we assume that $|\delta u_a^{\mathbf{k}}| \ll |u_a^{\mathbf{k}^s}|$. With the above approximation, we have following equation,

$$\left[\frac{d^2}{dt'^2} + \omega_\alpha^2 [1 - h_\alpha \cos(\gamma t')] \right] \delta\tilde{u}_\alpha^{\mathbf{k}}(t') = F_\alpha \cos(\gamma t') e^{-i\omega_\alpha t'} \; , \tag{6}$$

where $\alpha = \sigma$ or π. Here, we introduce the dimensionless variables $(\omega_\alpha, h_\alpha, F_\alpha)$. If F_σ (F_π) is negligible, then Eq. (6) is reduced to Mathieu's equation. In this case, the existence of the unstable solution for $\delta\tilde{u}_a^{\mathbf{k}}(t')$ may be expected. This phenomenon is well known as parametric resonance in classical mechanics. On the other hand, forced oscillation may be realized through the effect of F_σ (F_π), even if the model parameters do not allow for unstable regions for the parametric resonance. These phenomena are seen in the lowest and first excited pion modes, as demonstrated in the numerical calculations presented above.

Let us search the the unstable regions for Eq. (6). We perform the time integration over a certain interval for the magnitude of the quantum meson mode function, which means a kind of time average. Then, we compare the time integral at a certain time step with a proceeding one. If the value of this integral is larger than the last one for all time steps under consideration, then we decide that this mode is unstable.

Figure.3 shows the unstable region for quantum meson modes in the case of $F_\alpha = 3.0$. The vertical axis represents ω_α^2 and the horizontal axis represents $\omega_\alpha^2 h_\alpha/2$. The time integration is taken in 10 fm/c, in order to judge the stability of solution up to 100 fm/c. It is shown that the unstable regions become wider due to the effect of the forced oscillation and/or the beat induced by the oscillation of the chiral condensate, as compared with the parametric resonance only.
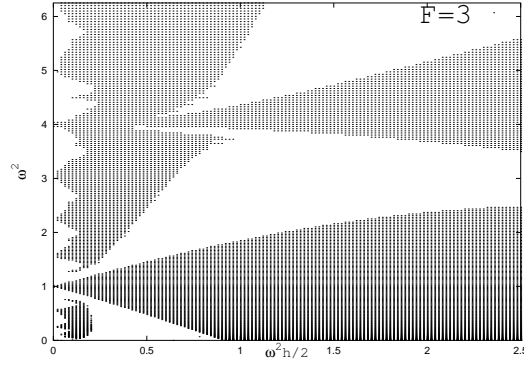
Fig. 3. The unstable regions for the solutions in Eq. (6) with $F_\alpha = 3.0$.
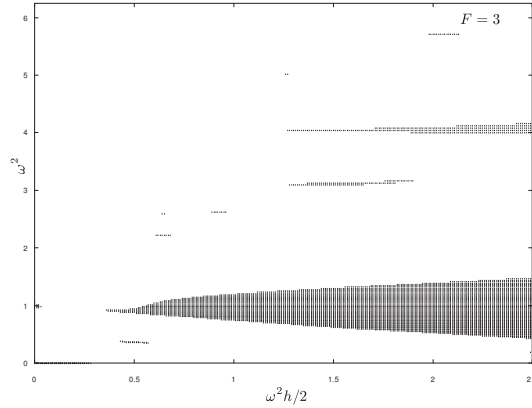
Fig. 4. The unstable regions for the solutions in Eq. (7) with $F_\alpha = 3.0$.

In the case with spatially expanding geometry, the time interval for the time integration to determine the unstable regions in our approach is taken as the rather short time 4 fm/c, because the unstable regions disappear when there is a strong damping effect for a long time. We calculate the time evolution up to 40 fm/c. The damping effect induced by the spatial expansion is introduced in the equations of motion for the quantum meson modes as follows:

$$\left\{ \frac{d^2}{dt'^2} + \omega_\alpha^2 [1 - h_\alpha \cos(\gamma t')e^{-\mu t'}] \right\} \delta \tilde{u}_\alpha^{[\mathbf{k}]}(t') = F_\alpha \cos(\gamma t')e^{-\mu t'}e^{-i\omega_\alpha t'} \ . \qquad (7)$$

In the case of $F_\alpha = 0$, the result for the unstable regions is well known, because this case corresponds to the case of the Mathieu equation with a friction term. Here, in Fig.4, we demonstrate the case of $F_\alpha = 3$ in Eq. (7). In this case, the forced oscillation and beat may realize. However, these effects only give a small

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modification. The reason for this is that the friction has a strong effect in this model. As a result, the amplification of the quantum meson modes with the spatially expanding geometry is not strong, although the amplification appears. These results imply that the effect of the forced oscillation or the beat ceases to be effective for the spatially expanding geometry.

4. Summary

We have demonstrated that the amplitude of quantum pion modes are amplified even in the late time of the chiral phase transition through the mechanism of forced oscillation or the beat, as well as the parametric resonance, in the framework of the O(4) linear sigma model. Also, we have determined the parameter regions in which the unstable solutions for the quantum meson mode functions exist. Further, we have found that the effect of forced oscillation ceases due to the strong friction in the case with the spatially expanding geometry.

   Of course, it is necessary to know the initial conditions in order to judge whether the amplification of the quantum meson mode occurs or not, that is, whether the parameters are in the unstable region or not, in realistic relativistic heavy ion collisions.

References

1. D. Vautherin and T. Matsui, *Phys. Rev.* **D55** (1997) 4492.
   D. Vautherin and T. Matsui, *Phys. Lett.* **B437** (1998) 137.
2. Y. Tsue, D. Vautherin and T. Matsui, *Prog. Theor. Phys.* **102** (1999) 313.
   Y. Tsue, D. Vautherin and T. Matsui, *Phys. Rev.* **D61** (2000) 076006.
3. K. Watanabe, Y. Tsue and S. Nishiyama, *Prog. Theor. Phys.* **113** (2005) 369.